**B.Sc. (Honours) Examination, 2018**
**Semester-III**
**Statistics**
**Course : CC-6A**
**(Statistical Inference)**

**Time : 3 Hours** **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions from the following

1. (a) Give the definition of a sufficient statistic. State Neyman-Fisher Factorization Theorem. 3

   (b) Let $T$ be a sufficient statistic for the parameter $\theta$ and let $\psi(T)$ be a one-to-one function of $T$. Show that $\psi(T)$ is also sufficient for $\theta$. 3

   (c) Let $X_1, X_2, \cdots, X_n$ be a random sample of size $n$ drawn from the following population. 4

$$f_\theta(x) = \frac{1}{\theta}e^{-x/\theta}, \; x > 0, \theta > 0.$$

   Obtain a sufficient statistic for $\theta$.

2. (a) State and prove Cramer-Rao inequality. When does the equality hold in this equality? 4

   (b) Show that an MVU estimator is unique, in the sense that if both $T_0$ and $T_1$ are MVU estimators, then $T_0 = T_1$ almost everywhere for all choices of the parameter. 3

   (c) Let $T$ and $U$ be consistent estimators of $\gamma(\theta)$ and $\delta(\theta)$ respectively. Show that $kT + lU$ is consistent for $k\gamma(\theta) + l\delta(\theta)$. 3

3. (a) Describe the estimation by the method of maximum likelihood. State and prove two of its important properties. 6

   (b) Let $X_1, X_2, \cdots, X_n$ be a random sample from a distribution with p.m.f

$$f_N(x) = \frac{1}{N}, \; x = 1, 2. \cdots, N; \; = 0, \; \text{if otherwise.}$$ 4

   Obtain the MLE of $N$.

4. (a) Describe the estimation by the method of moments. Show that, given independent random sample $X_1, X_2, \cdots, X_n$ from the population

$$f_\theta(x) = K(\theta)e^{-(\sum_{i=0}^{m}\theta_i x_i)}$$ 5

P.T.O.

the method of moments agrees with the method of maximum likelihood in estimating $\theta_i (i = 0, 1, \cdots, m)$.

(b) A random sample of size $n$ is drawn without replacement from a population of unknown size $N$, having $M$ (known) members of a specified type. If $x$ is the number of individuals of this type in the sample, obtain the MLE of $N$. 5

5. Distinguish between (1) Simple and Composite Hypotheses, (ii) Level and Size of a test, and (iii) UMP and UMPU tests. 10

6. State and prove Neyman-Pearson Lemma. Use this lemma to find UMP test for testing $H_0 : \mu = \mu_0$ against $\mu > \mu_0$, where $\mu$ is the mean of a normal population with known variance. 10